Short Note:
An Entropy-Based Approach to Establish MPS Templates

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The use of multiple-point statistics (MPS) shows great promise for characterizing data which display high-order structure such as curvilinearity or complex relations between facies. The use of MPS for facies modeling requires a template of points to be defined, within which the relevant statistics will be calculated and stored. This approach is used to minimize both the CPU time and memory requirements. Determining which arrangement of points to use in a MPS template is not trivial. Some points contain more relevant information; others contain significantly less, and at a certain distance from the central point some locations may not add any information at all. The methodology proposed here uses two-point entropy to quantify the "goodness" of points in a possible template.

Entropy of Categorical Data

The two-point entropy of a point *n* within a MPS template is defined as:

$$H_n = -\sum_{k=1}^{K}\sum_{k'=1}^{K} P_{kk'} \cdot \ln\left(P_{kk'}\right) \tag{1}$$

where K is the number of possible facies; and $P_{kk'}$ is the probability of facies k occurring at the central point and facies k' occurring at point n of the template. Higher entropy suggests more randomness and therefore less correlation between the central, estimated point and point n. Therefore, the points with the lowest entropy should be considered to contain the most information.

The central point of the template, which is the point to be estimated or perturbed using MPS, will always have the lowest possible entropy value; its two-point entropy is equal to the univariate entropy of the data. This value may be found by using the following properties:

$$P_{kk'} = P_k \text{ for all } k = k'$$

$$P_{kk'} = 0 \text{ for all } k \neq k'$$

Substituting these values into Equation 1,

$$H_{\min} = -\sum_{k=1}^{K} P_k \cdot \ln\left(P_k\right) \tag{2}$$

The maximum two-point entropy value may be found by considering that at some distance from the central point of the template, there is no additional information to be gained, randomness (and therefore entropy) is a maximum, and therefore the point *n* and the central point are entirely independent. In this case,

$$P_{kk'} = P_k \cdot P_{k'} \text{ for all } k \text{ and } k'$$

Substituting this into Equation 1,

$$H_{\max} = -\sum_{k=1}^{K}\sum_{k'=1}^{K} P_k \cdot P_{k'} \cdot \ln\left(P_k \cdot P_{k'}\right) \quad (3)$$

The numerical values of the minimum and maximum possible entropy values do not necessarily have any meaning for a given template; however, they could be used to standardize the entropy values between 0 and 1, or to define a cutoff for which points should not be included (say, a standardized value of 0.7 or greater).

While a two-point entropy does not necessarily account for all of the complex information contained in MPS, using a full *N*-point entropy would be very time-consuming and cumbersome; with *K* facies and *N* points in a potential arrangement of points within the template, there are K^N different MPS histogram classes that would have to be calculated. This process would then need to be repeated for the next possible arrangement of *N* points and the entropies compared; and so on for many arrangements. For three or four points this would not be an issue, but as the template grows to as many as a hundred points or more using the full entropy is no longer feasible.

Examples

The attached figures show eight examples using entropy to find the best points for a MPS template. Each example shows the training image from which the entropies were calculated, a map of the entropy of points in a 21x21 template (maximum offsets of 10 in each direction), and a graph of the entropies of the points, sorted from lowest entropy to greatest. Some of the notable features in the examples are: (1) In cases where there is repetition of a pattern, the entropy maps reflect this feature; (2) When a single structure dominates the training image the shape of the entropy "shell" appears ellipsoidal, coinciding with the direction of major continuity; (3) When several major structures are clearly visible they are all reflected in the entropy map; (4) With the points sorted from lowest to highest entropy, the graphs are obviously increasing. The graphs all approach the maximum entropy value asymptotically; (5) Departing from point 1, the central point with the lowest possible entropy, the entropy of surrounding points increases quite quickly before leveling off near the maximum value.

References

C.V. Deutsch. *Annealing Techniques Applied to Reservoir Modeling and the Integration of Geological and Engineering (Well Test) Data*. Ph.D. Thesis, Stanford University, 1992.

M.J. Pyrcz and C.V. Deutsch. A Library of Training Images for Fluvial and Deepwater Reservoirs and Associated Code. In *Centre for Computational Geostatistics*, Report Five, 2003.

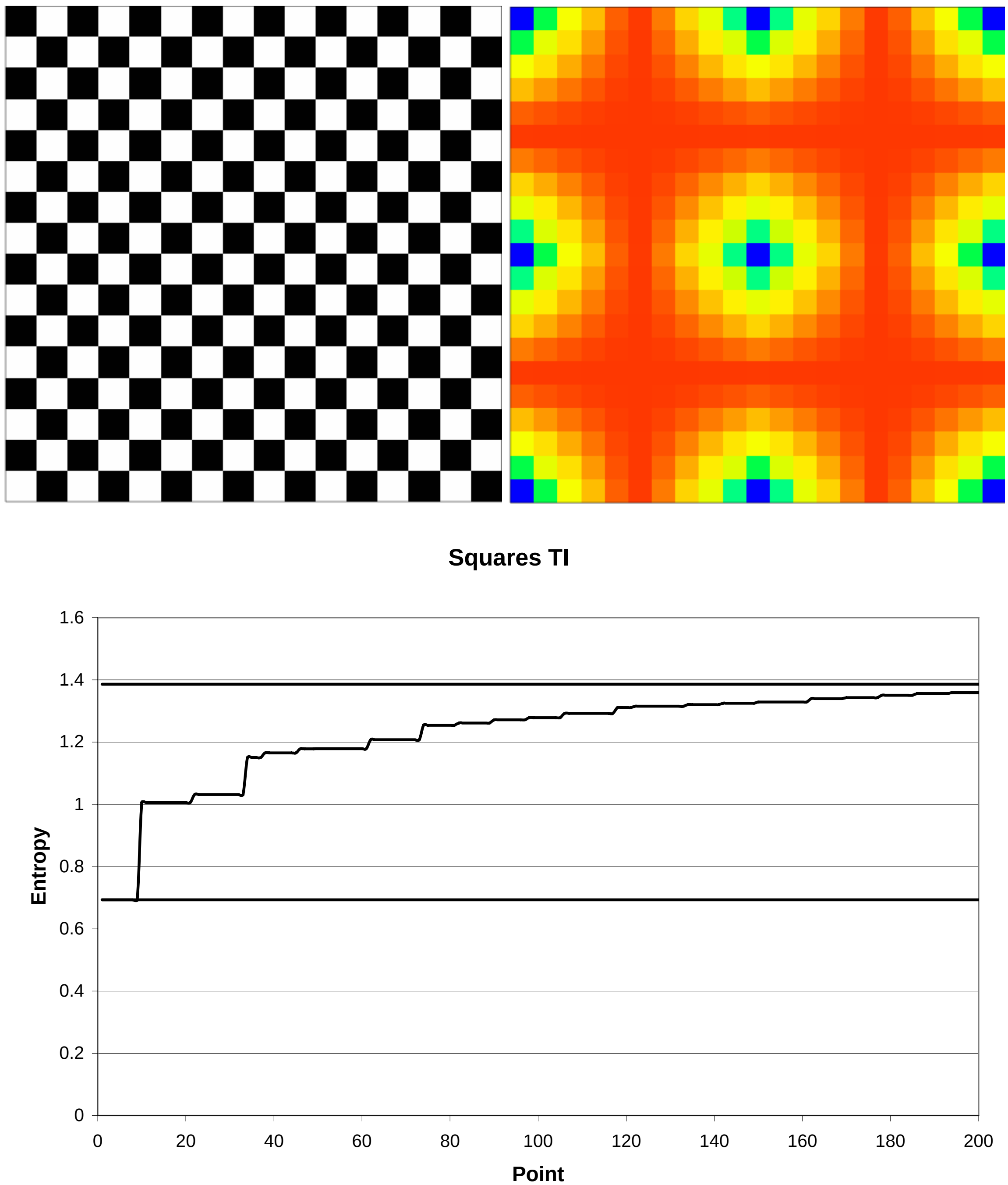

Figure 1: Training image made up entirely of 10x10 squares; map of entropy for a 21x21 statistical template; graph of entropy of points sorted from minimum to maximum.

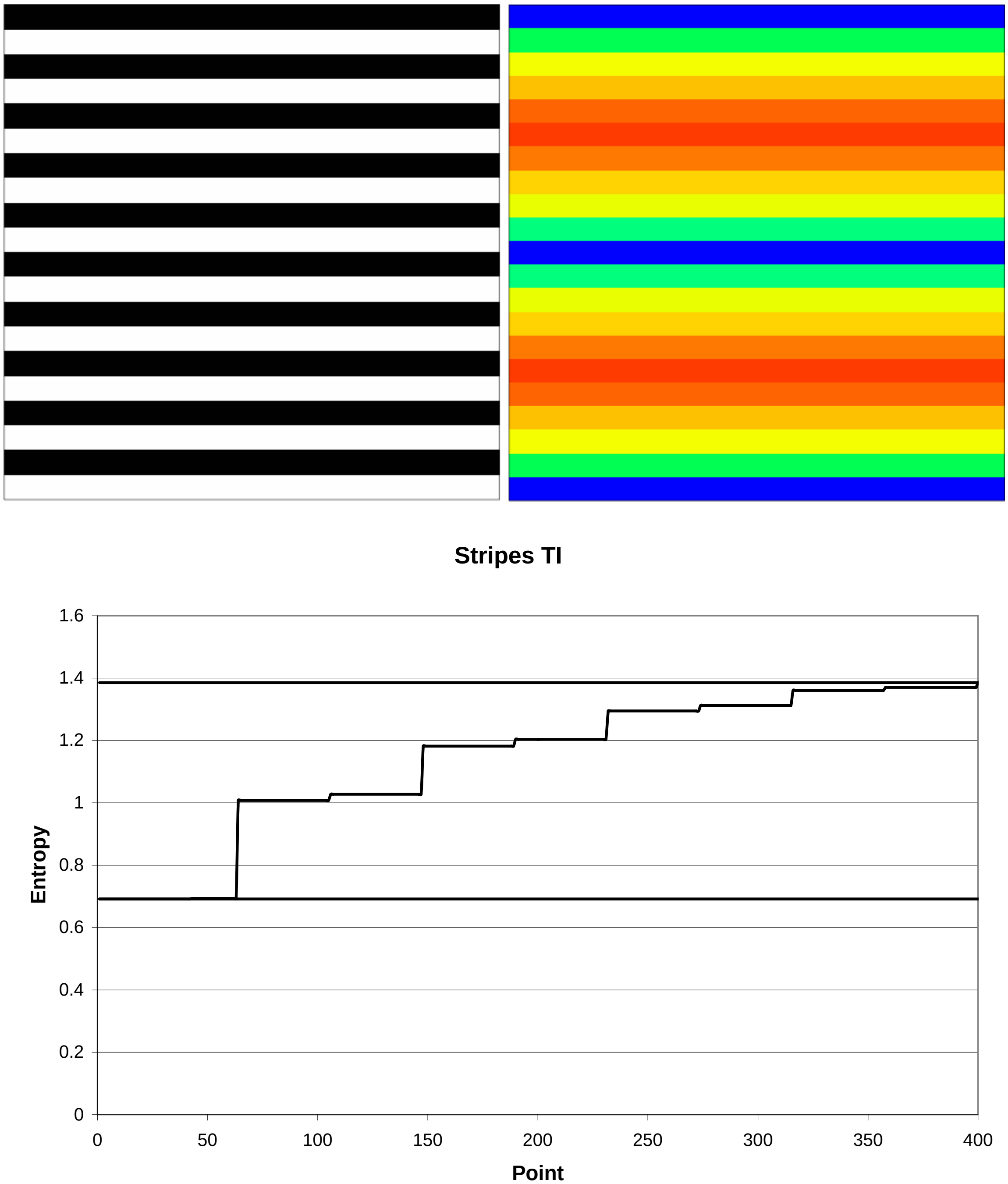

Figure 2: Training image made up entirely stripes of thickness 10; map of entropy for a 21x21 statistical template; graph of entropy of points sorted from minimum to maximum.

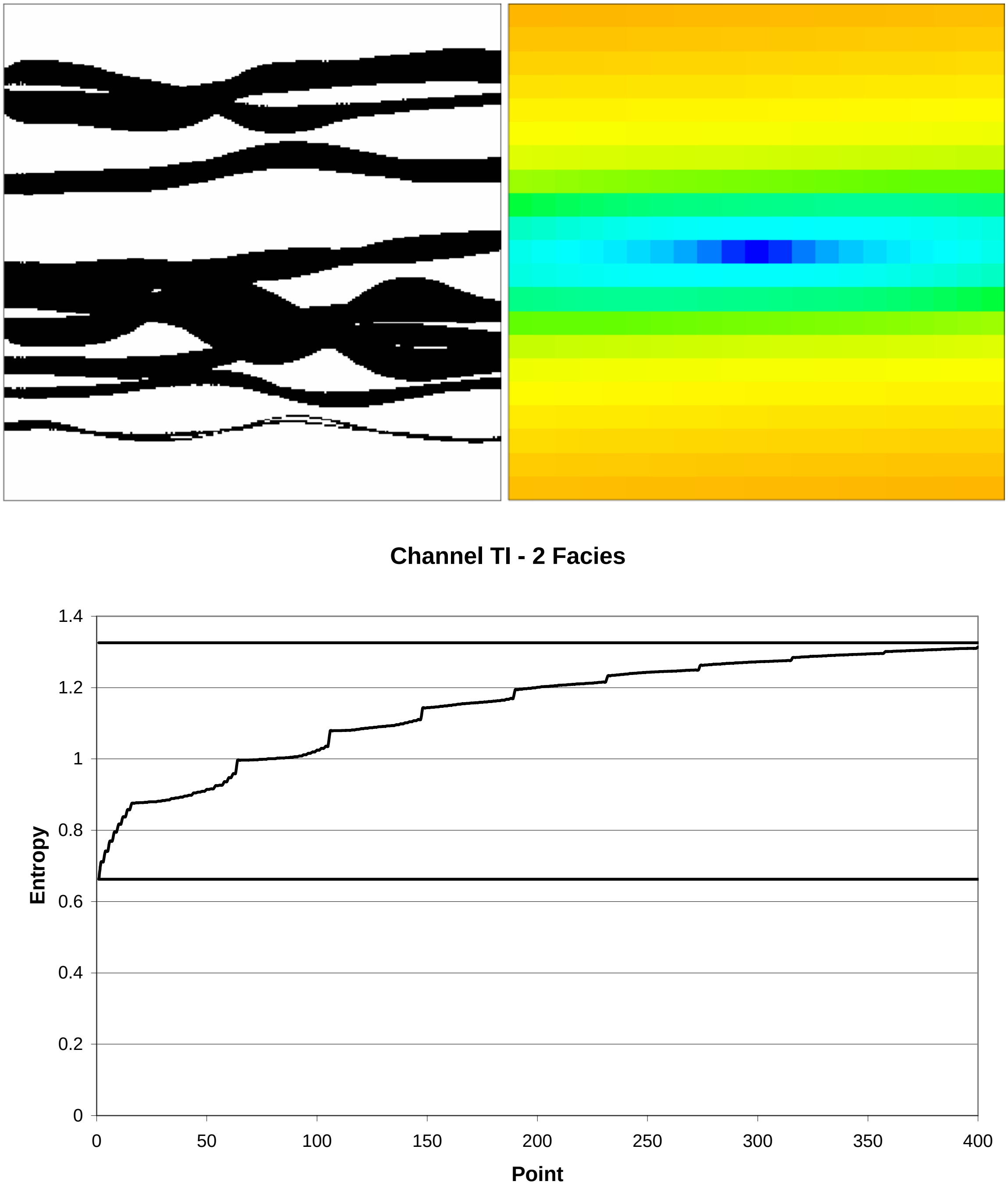

Figure 3: Training image made up of channel and background facies; map of entropy for a 21x21 statistical template; graph of entropy of points sorted from minimum to maximum. (TI from Pyrcz and Deutsch, 2003)

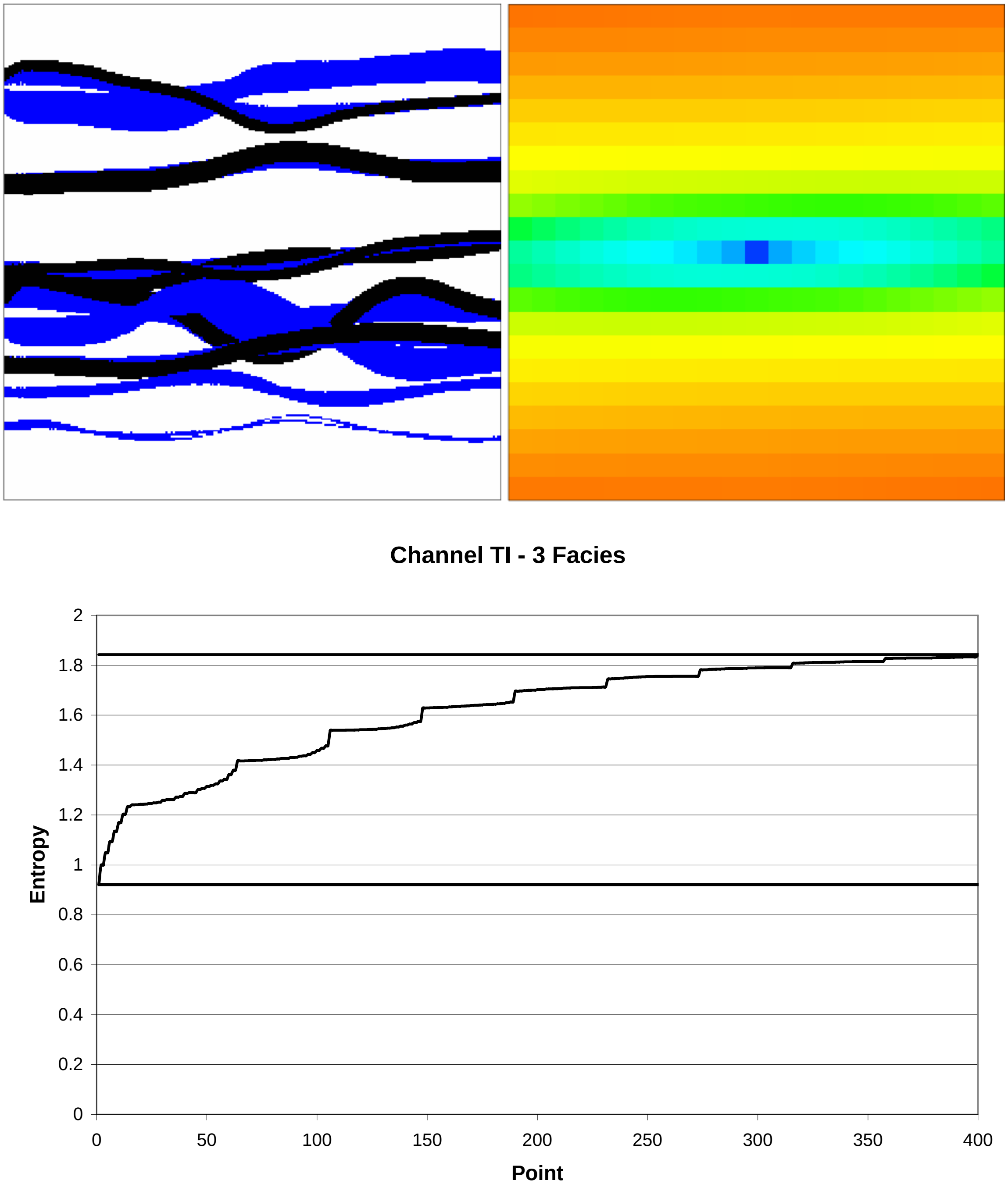

Figure 4: Training image made up of two different channels and background facies; map of entropy for a 21x21 statistical template; graph of entropy of points sorted from minimum to maximum. (TI from Pyrcz and Deutsch, 2003)

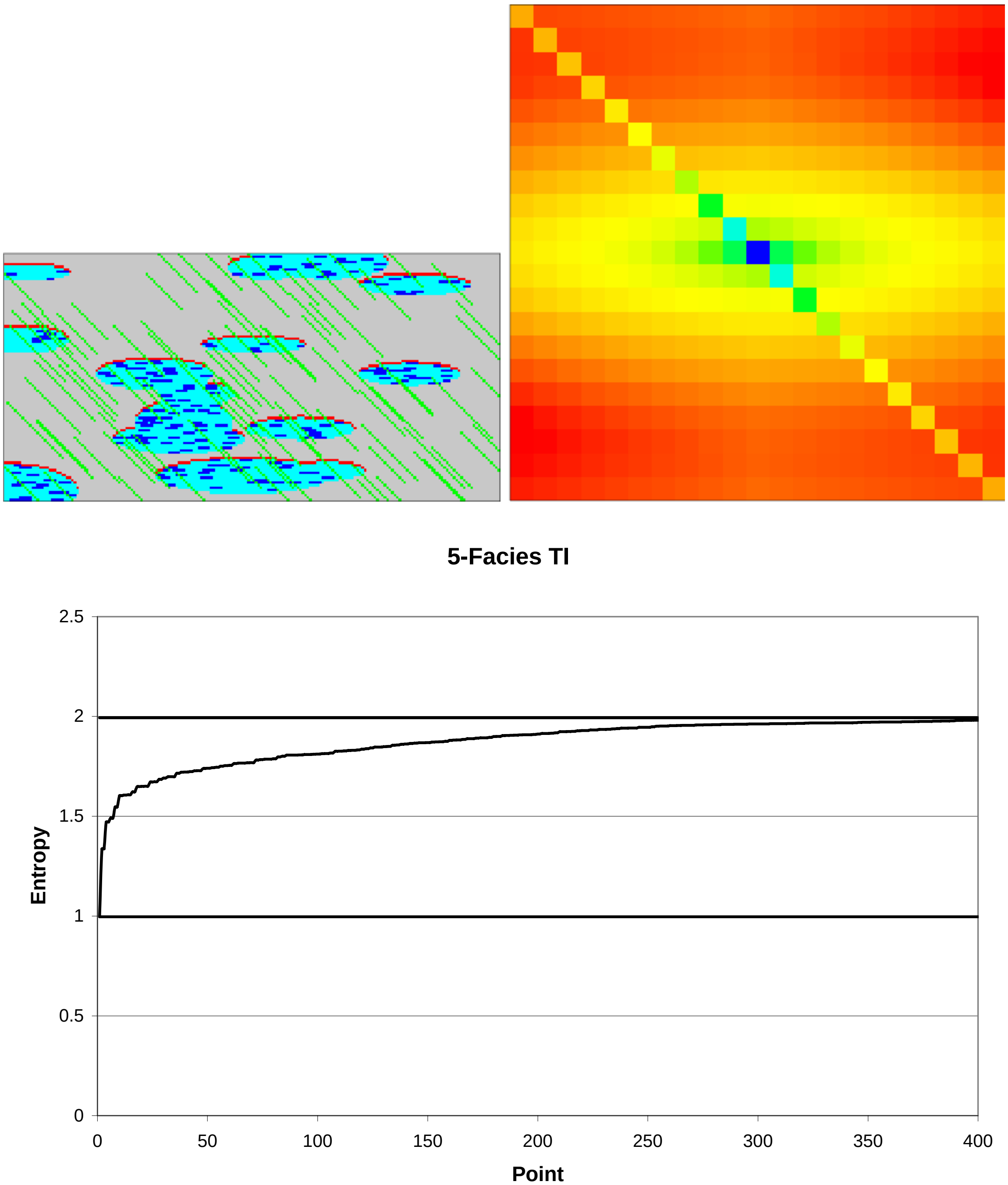

Figure 5: Training image containing five facies with complex relations; map of entropy for a 21x21 statistical template; graph of entropy of points sorted from minimum to maximum. (TI From Deutsch, 1992)

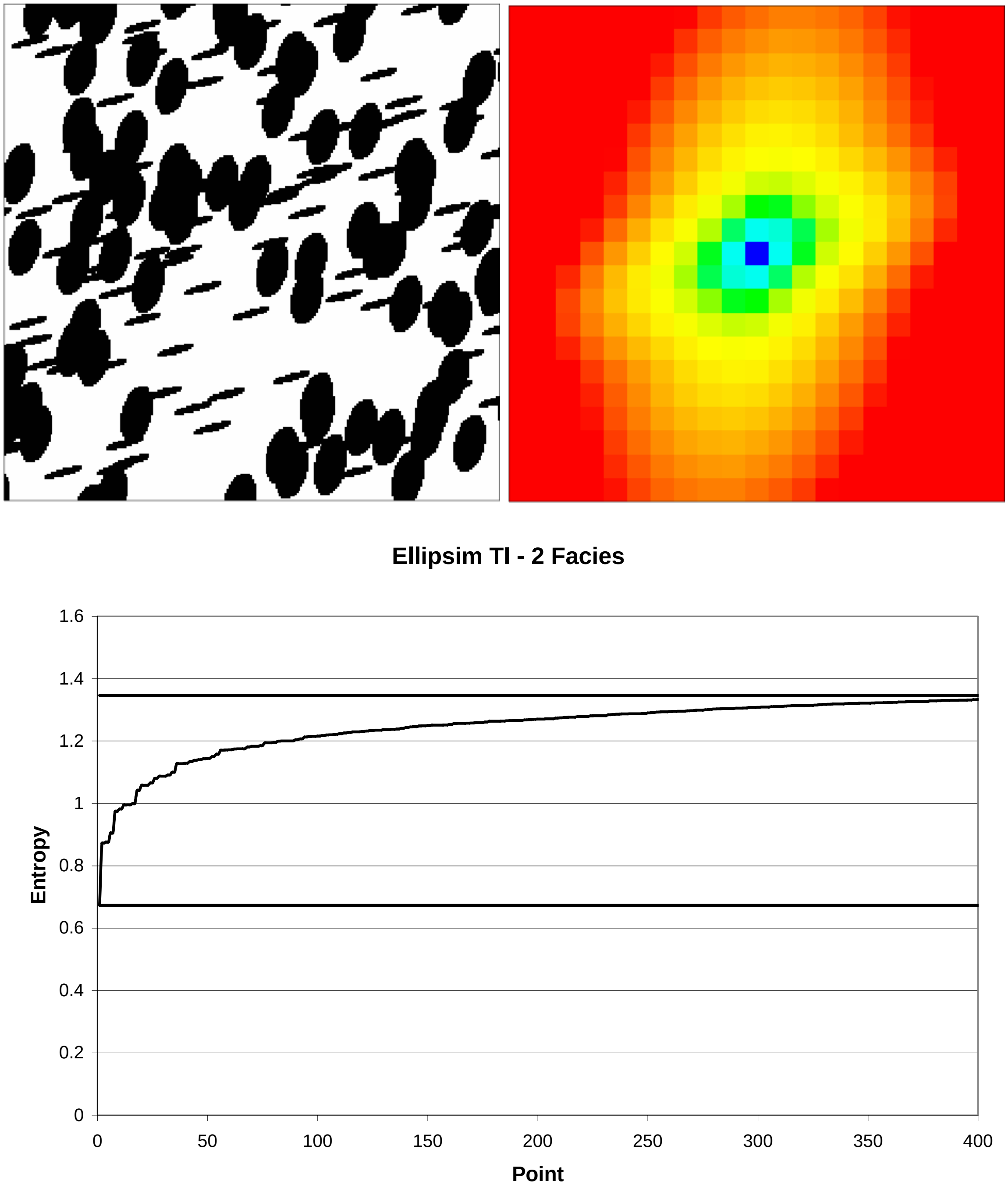

Figure 6: Training image containing ellipses of two sizes and orientations; map of entropy for a 21x21 statistical template; graph of entropy of points sorted from minimum to maximum.

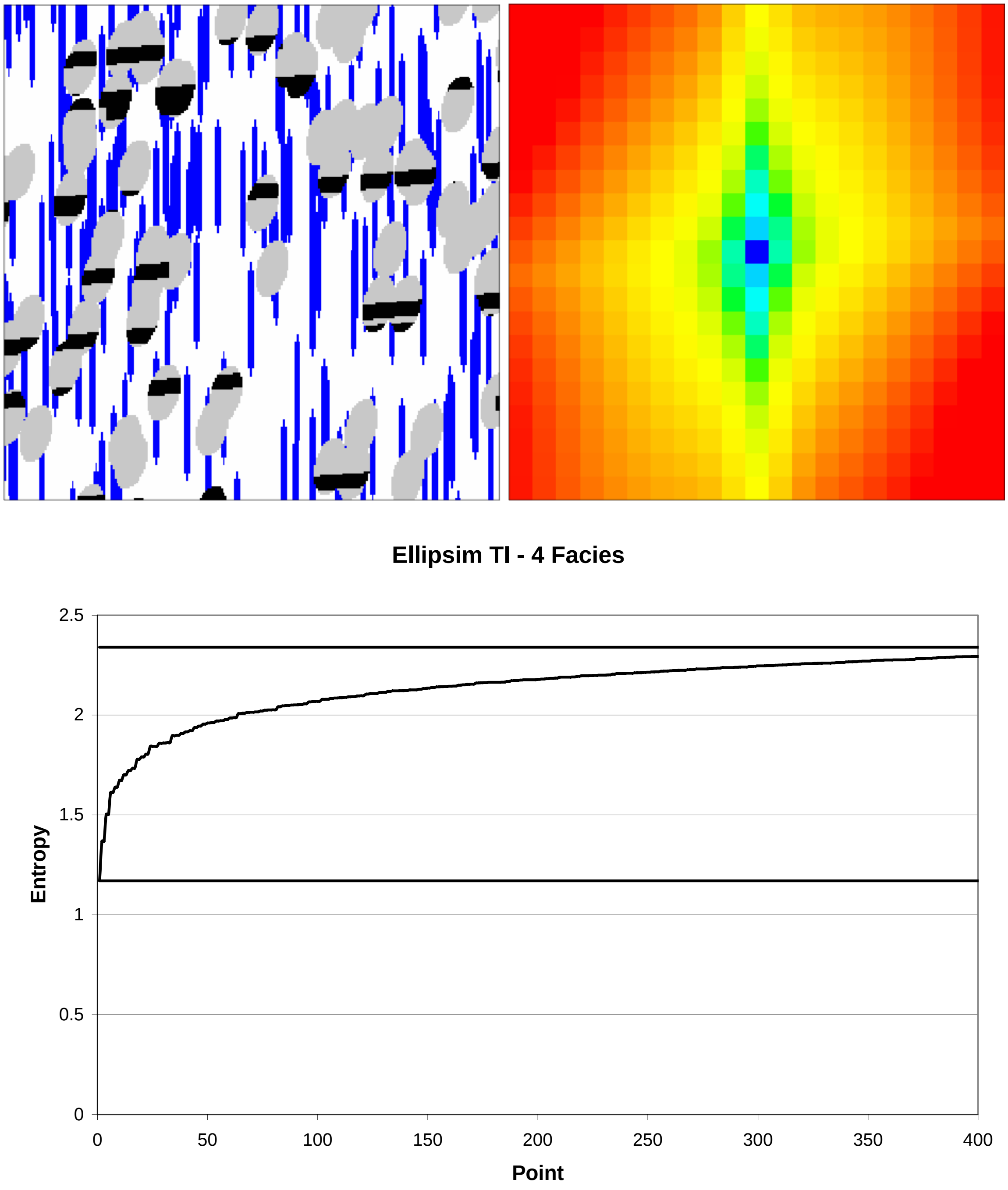

Figure 7: Training image containing ellipses of several sizes and orientations, with complex facies relations; map of entropy for a 21x21 statistical template; graph of entropy of points sorted from minimum to maximum.

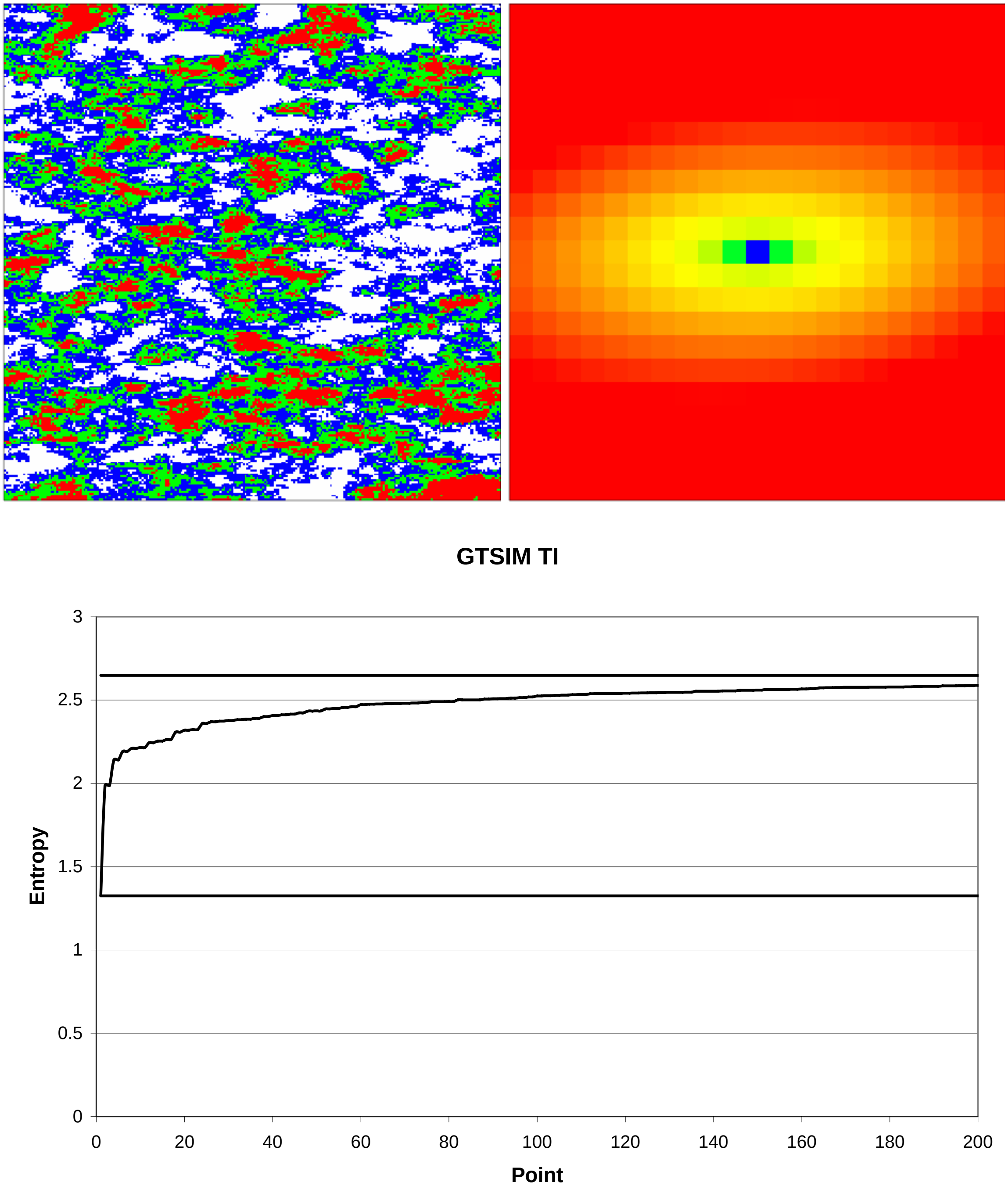

Figure 8: Training image with four facies created using truncated Gaussian simulation; map of entropy for a 21x21 statistical template; graph of entropy of points sorted from minimum to maximum.